

# STATISTICAL SEPARABILITY AND THE CONSISTENCY BETWEEN QUANTUM THEORY, RELATIVITY AND THE CAUSALITY

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We show that the non-locality together with the statistical character makes the world statistically separable. The super-luminal signal transmission is impossible. The quantum theory is therefore consistent with the relativity and the causality.

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In 1935, Einstein *et al*[1] assumed that the world is separable to prove the incompleteness of quantum theory. The separability here means one can always separate two systems by a space, so that a disturbance act on one of them cannot influence another system immediately. This separability seemed to be self-evident, since otherwise the prompt signal transmission would be possible. About thirty years later, Bell[2] changed the thought experiment used in [1] to be a kind of experiment which can be really done. After that, many experimental results on the problem of separability have been reported. They show that Einstein's assumption of separability is false. One may see this conclusion in examples [3, 4]. The world is proven to be non-separable in Einstein's sense. This result raises the question on the consistency between the quantum theory, relativity, and the causality, and on the possibility of the super-luminal transmission of signals.

The quantum state of a system is usually defined at a given time but in the whole space, therefore is non-local and non-separable, in contrast to the classical state. Two parts of a system even though separated by empty space may be correlated. Their states may be entangled if they had contacted and therefore interacted each other previously. People tried to use this entanglement for signal transmission, and have got experimental success[5]. It is called quantum communication. However the super-luminal signal transmission is still not realized, and it has been proven to be impossible[6]. It makes people feel that the quantum theory may be consistent with relativity and causality. In the following we show that this feeling is reliable.

To make the theory be Lorentz invariant, the quantum state is defined on a space-like super-surface of space-time in general. It is non-local. However, the dynamics of the state evolution along a time-like direction is governed by differential equation. It is therefore local. According to the multi-time formulation[7]-[9] of the quantum field theory, the state  $|\sigma\rangle$  on a space-like super-surface  $\sigma$  and the state  $|\sigma'\rangle$  on a nearby space-like super-surface  $\sigma'$  are

related by the Schrödinger equation

$$i\hbar \frac{\partial |\sigma\rangle}{\partial \sigma} = -T_{\mu\nu} n_\mu n_\nu |\sigma\rangle. \quad (1)$$

Here, the differential  $d\sigma$  denotes the infinitesimal 4-dimensional space-time volume between super-surfaces  $\sigma'$  and  $\sigma$  around a space-time point.  $(T_{\mu\nu})$  is the energy-momentum tensor operator of the system and  $(n_\mu)$  is the time-like unit normal 4-vector of the super-surface  $\sigma$ , both at this point. The solution of (1) may be written in the form

$$|\sigma\rangle = U(\sigma, \sigma_0) |\sigma_0\rangle, \quad (2)$$

with

$$i\hbar \frac{\partial U(\sigma, \sigma_0)}{\partial \sigma} = -T_{\mu\nu} n_\mu n_\nu U(\sigma, \sigma_0), \quad (3)$$

$\sigma_0$  is an arbitrarily fixed space-like super-surface. The differential equation shows the evolution of the state from point to point. The relativity and causality require that a disturbance of the state  $|\sigma_0\rangle$  at the point  $(\mathbf{r}_0, t_0)$  on it may influence the properties of state  $|\sigma\rangle$  at those points on it only, which are inside the light cone with its vertex at  $(\mathbf{r}_0, t_0)$ . Properties of the state  $|\sigma\rangle$  at the point  $(\mathbf{r}, t)$  on  $\sigma$  outside this light cone do not respond to this disturbance.  $U(\sigma, \sigma_0)$  is therefore a matrix (or an integral operator), only those rows and columns have non-zero non-diagonal elements, which are related to the points inside this light cone. The remaining part of the matrix (or the integral operator) is unit.

A state of the system is determined by a complete measurement. For a macroscopic system, the measurement is usually incomplete. In this case, a density operator, instead of a state, is determined. The density operator is also defined on a space-like super-surface. Its time evolution is governed by the von Neumann equation

$$\frac{d\rho}{d\sigma} = -c [T_{\mu\nu}, \rho] n_\mu n_\nu, \quad (4)$$

in which  $[A, B] \equiv (AB - BA)/i\hbar$  is the quantum Poisson bracket of  $A$  and  $B$ . The solution of this equation is

$$\rho(\sigma) = U(\sigma, \sigma_0) \rho(\sigma_0) U(\sigma_0, \sigma). \quad (5)$$

Since the super-surface  $\sigma$  is space-like, dynamical variables on different points of it commute with each other. They belong to different degrees of freedom. Taking the direct products of the eigenstates of commuting dynamical variables at different points on  $\sigma$  as bases, one may expand the state of the system. Although the bases in the state are entangled in general, one can still measure the local property of the system. The local property of the system at the point  $\mathbf{r}$  on  $\sigma$  is described by the reduced density operator

$$\rho(\mathbf{r}) = \text{Tr}_{\bar{\mathbf{r}}} \rho(\sigma) . \quad (6)$$

The subscript  $\bar{\mathbf{r}}$  in (6) denotes that the trace is a sum of matrix elements which are diagonal respect to the degrees of freedom other than those at the point  $\mathbf{r}$  only. We show in the following that if  $(\mathbf{r}, t)$  is outside the light cone of the point  $(\mathbf{r}_0, t_0)$ , the disturbance at  $(\mathbf{r}_0, t_0)$  cannot influence the reduced density matrix  $\rho(\mathbf{r}, t)$ , therefore cannot influence the result of the local measurement at point  $(\mathbf{r}, t)$ . We call this property the statistical separability.

Denote the degrees of freedom in a macroscopically infinitesimal neighborhood of the point  $\mathbf{r}$  by  $a$ , and all other degrees of freedom by  $b$ . The bases to be used in expanding the state of the system are  $[|n_a, n_b\rangle \equiv |n_a\rangle|n_b\rangle]$ .  $n_a$  is a complete set of quantum numbers for degrees  $a$  of freedom, while  $n_b$  is that for degrees  $b$  of freedom. In this representation, we may write

$$\rho(\sigma_0) \equiv \sum_{n_a, n_b, n'_a, n'_b} |n'_a\rangle\langle n'_b| \rho_{n'_a n'_b; n_a n_b} \langle n_a| \langle n_b| , \quad (7)$$

$[\rho_{n'_a n'_b; n_a n_b}]$  are the matrix elements of  $\rho(\sigma_0)$ . According to the argument after (3), if  $(\mathbf{r}, t)$  is outside the light cone of the point  $(\mathbf{r}_0, t_0)$ , we may write

$$U(\sigma, \sigma_0) \equiv \sum_{n_a, n_b, n'_b} |n_a\rangle\langle n'_b| U_{n'_b, n_b} \langle n_a| \langle n_b| , \quad (8)$$

and

$$U(\sigma_0, \sigma) \equiv \sum_{n_a, n_b, n'_b} |n_a\rangle\langle n'_b| U_{n_b, n'_b}^* \langle n_a| \langle n_b| . \quad (9)$$

The relation  $U(\sigma_0, \sigma)U(\sigma, \sigma_0) = 1$  requires

$$\sum_{n'_b} U_{n'_b n_b} U_{n'_b n_b}^* = \delta_{n'_b, n_b} . \quad (10)$$

Substituting (7-9) into (5), then substituting the result into (6), by use of (10) and the ortho-normality of base states  $[|n_a\rangle|n_b\rangle]$  we obtain

$$\begin{aligned} \rho(\mathbf{r}) &= \sum_{n_a n'_a n_b n'_b} |n'_a\rangle U_{n_b, n'_b} \rho_{n'_a n'_b; n_a n_b} U_{n_b, n'_b}^* \langle n_a| \\ &= \sum_{n_a n'_a n'_b} |n'_a\rangle \rho_{n'_a n'_b; n_a n'_b} \langle n_a| = \rho_0(\mathbf{r}) , \end{aligned} \quad (11)$$

in which

$$\rho_0(\mathbf{r}) \equiv \sum_{n_a n'_a n_b} |n'_a\rangle \rho_{n'_a n_b; n_a n_b} \langle n_a| \equiv \text{Tr}_{\bar{\mathbf{r}}} \rho(\sigma_0) \quad (12)$$

is the reduced density operator at point  $\mathbf{r}$  on the super-surface  $\sigma_0$ . Since the local reduced density operator is the complete description of the local measurement in quantum theory, (11) shows that the result of local measurement at a point does not change until the point goes into the light cone of the disturbance. This is the statistical separability of the space. One may immediately conclude that although the quantum states are non-local and local states at two points separated by space-like distance may be entangled (non-separable), the communication between two space-likely separated points (super-luminal communication) is still impossible by disturbance and measurement on a quantum system. The causality in relativity is therefore ensured by the present quantum theory.

The argument for the possibility of super-luminal communication by use of the non-locality of the quantum state is based on an assumption that the relation between the disturbance and the measurement result is deterministic. This is not true in quantum theory. According to quantum theory, disturbance changes the quantum state, and therefore changes the statistical distribution of the measurement results. The deterministic theory with non-local state contradicts the relativity and the causality. Fortunately, in quantum theory, one works not only with non-local state but also with original statistics. This two wings make the quantum theory be consistent with relativity and causality.

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